Solutions

Math 2D Quiz 5 Afternoon - February 18th

Please put name and ID on *both* sides for grading and redistribution!

Show all of your work. *There is a question on the back side.

1. Consider the surface equation $x + 2y + 3z = e^{xyz}$. Let $P = (1, 0, 0)$.

(a) Find the equation of the tangent plane at P.

(b) Find the symmetric equation of the normal line to the surface at P.

(c) Find (using Implicit Differentiation / Implicit Function Theorem):

$$\frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}, \quad \text{and (be careful with this one)} \quad \frac{\partial x}{\partial y}.$$

Your answers for (c) are functions - I do NOT want you to evaluate them at the point P.

a) First, $F(x,y,z) = x + 2y + 3z - e^{xyz} = 0$.

Then $\nabla F(x,y,z) = \langle 1 - yze^{xyz},\ 2 - xze^{xyz},\ 3 - xye^{xyz} \rangle$. +1

So, $\nabla F(1,0,0) = \langle 1, 2, 3 \rangle$.

Thus, the tangent plane eqn is: $\boxed{(x-1) + 2y + 3z = 0}$ +1

b) From this, the normal line to the surface at P is: $\boxed{(x-1) = \frac{y}{2} = \frac{z}{3}}$ +1

c)

$$\frac{\partial z}{\partial x} = - \frac{F_x}{F_z} = - \frac{1 - yze^{xyz}}{3 - xye^{xyz}}$$

$$\frac{\partial z}{\partial y} = - \frac{F_y}{F_z} = - \frac{2 - xze^{xyz}}{3 - xye^{xyz}}$$

$$\frac{\partial x}{\partial y} = - \frac{F_y}{F_x} = - \frac{2 - xze^{xyz}}{1 - yze^{xyz}}$$

Use F_x, F_y, F_z from part (a)!

+2

2. Let $f(x,y) = e^{xy}\cos(x^2+y^2)$.
(a) Compute $\nabla f(x,y)$.
(b) Compute $D_{\mathbf{u}} f(0, \sqrt{\frac{\pi}{2}})$ in the direction of $\mathbf{u} = < 1, 1 >$. ($\hat{\mathbf{u}} = < \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$).
(c) This function was originally written as

$$f(r,\theta) = e^r\cos(\theta), \quad r = st, \quad \theta = s^2 + t^2.$$

Use the chain rule to compute

$$\frac{\partial f}{\partial s} \quad \text{and} \quad \frac{\partial f}{\partial t}$$

<u>without</u> plugging in for r, θ as functions of s, t (like we did in section).

a) $\nabla f(x,y) = < f_x(x,y) \, , \, f_y(x,y) >$

$= < ye^{xy}\cos(x^2+y^2) - 2xe^{xy}\sin(x^2+y^2) \, ; \, xe^{xy}\cos(x^2+y^2) - 2ye^{xy}\sin(x^2+y^2) >$

+1 +1

b) First, $\nabla f(0, \sqrt{\frac{\pi}{2}}) = < 0 - 0 \, , \, 0 - 2\sqrt{\frac{\pi}{2}}\, e^0 \sin(\frac{\pi}{2}) >$

$= < 0, -\sqrt{2\pi} >$

Thus, $D_u f(0, \sqrt{\frac{\pi}{2}}) = < 0, -\sqrt{2\pi} > \cdot < \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$

$= \boxed{-\sqrt{\pi}}$ +1

c) $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial r} \cdot \frac{\partial r}{\partial s} + \frac{\partial f}{\partial \theta} \cdot \frac{\partial \theta}{\partial s} = \boxed{e^r\cos\theta \cdot t \ominus e^r\sin\theta \cdot 2s}$ +1

$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial r} \cdot \frac{\partial r}{\partial t} + \frac{\partial f}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} = \boxed{e^r\cos\theta \cdot s \ominus e^r\sin\theta \cdot 2t}$ +1